UUCMS. No.

B.M.S COLLEGE FOR WOMEN
BENGALURU – 560004

I SEMESTER END EXAMINATION – APRIL 2024

M.Sc. MATHEMATICS - ELEMENTARY NUMBER THEORY
(CBCS Scheme-F+R)

Course Code: MM107S | **QP Code: 11006**
Duration: 3 Hours | **Max. Marks: 70**

Instructions: 1) All questions carry equal marks.
2) Answer any five full questions.

1. (a) State and prove Euclidean algorithm.
   (b) Prove that the linear Diophantine equation $ax + by = c$ has a solution if and only if $d|c$ where $d = gcd(a, b)$ and in this case there are infinitely many solutions.
   (7+7)

2. (a) Prove that if P_n is the nth prime then $P_n \leq 2^{2^{n-1}}$ for all $n \geq 1$.
   (b) Prove that there are infinitely many primes of the form $4q + 3$.
   (7+7)

3. (a) Show that 41 divides $2^{20} - 1$.
   (b) Find the remainder obtained upon dividing the sum $1! + 2! + 3! + \cdots + 99! + 100!$ by 12.
   (c) Find the remainders when 2^{50} and 41^{65} are divided by 7. **(5+4+5)**

4. (a) State and prove Wilson's theorem. Find the remainder when $97!$ is divided by 101.
   (b) If $ca \equiv cb\ (mod n)$ then prove that $a \equiv b\ \left(mod \left(\frac{n}{d}\right)\right)$ where $d = \gcd(c, n)$.
   (8+6)

5. (a) Let p be an odd prime and m and n are integers such that $(m, p) = 1$ and $(n, p) = 1$ then prove that $\left(\frac{mn}{p}\right) = \left(\frac{m}{p}\right)\left(\frac{n}{p}\right)$.
   (b) State and prove Gauss lemma.
   (7+7)

6. (a) State and prove Quadratic reciprocity law for Legendre symbol
   (b) Compute $(31/103)$.
   (c) Compute the Jacobi symbol $71/375$.
   (8+3+3)

7. (a) Prove that a positive integer n is a sum of two squares if and only if every prime, $q \equiv 3(mod\ 4)$ divides n to an even power.

   (b) Express 221 and 6409 as sums of two squares.

   (c) Define Pythagorean triple. Prove that if x, y, z is a primitive Pythagorean triple then one of the integers $x \& y$ is even while the other is odd.

(7+3+4)

8. (a) Prove that an odd prime p is expressible as sum of two squares if and only if $p \equiv 1(mod 4)$.

   (b) State and prove Fermat's last theorem. **(6+8)**

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